



Prototype Example – CALIFORNIA MANUFACTURING COMPANY (HL¹, §1.1, pg. 465)

The CALIFORNIA MANUFACTURING COMPANY is considering expansion by building a new factory in either Los Angeles (LA) or San Francisco (SF), or perhaps even in both cities. It also is considering building at most one new warehouse, but the choice of a location is restricted to a city where a new factory is being built. The *net present value* (total profitability considering the time value of money) of each of these alternatives is shown in the second column of the table below. The right most column gives the capital required (already included in the net present value) for the respective investments, where the total capital available is \$10 million. The objective is to find the feasible combination of alternatives that maximizes the total net present value.

Build a	Net Present Value	Capital Requirement
Factory in LA	\$9 million	\$6 million
Factory in SF	\$5 million	\$3 million
Warehouse in LA	\$6 million	\$5 million
Warehouse in SF	\$4 million	\$2 million

ILP Exercises

43. TBA Airlines is a small air company, specialized in regional flights. The management is considering an expansion and it has the possibility to buy small or medium size airplanes. Find the best strategy, knowing that at the moment no more than two small airplanes can be bought and that \$100 millions are available to invest. Consider also the values in the following table:

	small airplane	medium size airplane
annual profit per airplane	\$1 million	\$5 millions
cost per airplane	\$5 millions	\$50 millions

44. A company has a portfolio with three investment projects with a life time of 4 years. The table below presents the *cash-flows* and the corresponding net present values (NPV) at 10% year rate:

Project \	year 0	year 1	year 2	year 3	NPV (m.u.)
A	-100	50	300	-20	178
B	-300	100	150	200	65
C	0	-100	75	200	121

Project **B** can be deferred by one year and in that case the NPV is 59 m. u., as it can be verified. There are budget constraints in the first two years of 300 and 200 m. u., respectively. Formulate this problem and solve it by the *solver/excel*.

45. An oil company intends to select 5 out of 10 wells: **P1, P2, ..., P10**, to which are associated the costs c_1, c_2, \dots, c_{10} , respectively. According to commitments with the local government, the company must comply with the following restrictions for regional development:
- r1) the selection of both **P1** and **P7** block selection of **P8**;
 - r2) the selection of **P3** or **P4** block selection of **P5**;
 - r3) from **P5, P6, P7** and **P8** at most two can be selected;
 - r4) the selection of **P1** forces selection of **P10**.

Formulate the problem and solve it assigning costs at your choice.

46. Solve problems a) 11.1-2; b) 11.1-3., c) 11.1-4., d) 11.3-1 of HL¹. (pg. 524-527)



- a) 11.1-2* A young couple, Eve and Steven, want to divide their main household chores (marketing, cooking, dishwashing, and laundering) between them so that each has two tasks but the total time they spend on household duties is kept to a minimum. Their efficiencies on these tasks differ, where the time each would need to perform the task is given by the following table:

	Time Needed per Week			
	Marketing	Cooking	Dishwashing	Laundry
Eve	4.5 hours	7.8 hours	3.6 hours	2.9 hours
Steven	4.9 hours	7.2 hours	4.3 hours	3.1 hours

- (a) Formulate a BIP model for this problem.
(b) Display this model on an Excel spreadsheet.
(c) Use the computer to solve this model.

- d) 11.3-1.* The Research and Development Division of the Progressive Company has been developing four possible new product lines. Management must now make a decision as to which of these four products actually will be produced and at what levels. Therefore, an operations research study has been requested to find the most profitable product mix.

A substantial cost is associated with beginning the production of any product, as given in the first row of the following table. Management's objective is to find the product mix that maximizes the total profit (total net revenue minus start-up costs).

	Product			
	1	2	3	4
Start-up cost	\$50,000	\$40,000	\$70,000	\$60,000
Marginal revenue	\$ 70	\$ 60	\$ 90	\$ 80

Let the continuous decision variables x_1 , x_2 , x_3 , and x_4 be the production levels of products 1, 2, 3, and 4, respectively. Management has imposed the following policy constraints on these variables:

- No more than two of the products can be produced.
- Either product 3 or 4 can be produced only if either product 1 or 2 is produced.
- Either $5x_1 + 3x_2 + 6x_3 + 4x_4 \leq 6,000$
or $4x_1 + 6x_2 + 3x_3 + 5x_4 \leq 6,000$.

- (a) Introduce auxiliary binary variables to formulate a mixed BIP model for this problem.
(b) Use the computer to solve this model.

- b) 11.1-3. A real estate development firm, Peterson and Johnson, is considering five possible development projects. The following table shows the estimated long-run profit (net present value) that each project would generate, as well as the amount of investment required to undertake the project, in units of millions of dollars.

	Development Project				
	1	2	3	4	5
Estimated profit	1	1.8	1.6	0.8	1.4
Capital required	6	12	10	4	8

The owners of the firm, Dave Peterson and Ron Johnson, have raised \$20 million of investment capital for these projects. Dave and Ron now want to select the combination of projects that will maximize their total estimated long-run profit (net present value) without investing more than \$20 million.

- (a) Formulate a BIP model for this problem.
(b) Display this model on an Excel spreadsheet.
(c) Use the computer to solve this model.

- c) 11.1-4. The board of directors of General Wheels Co. is considering six large capital investments. Each investment can be made only once. These investments differ in the estimated long-run profit (net present value) that they will generate as well as in the amount of capital required, as shown by the following table (in units of millions of dollars):

	Investment Opportunity					
	1	2	3	4	5	6
Estimated profit	15	12	16	18	9	11
Capital required	38	33	39	45	23	27

The total amount of capital available for these investments is \$100 million. Investment opportunities 1 and 2 are mutually exclusive, and so are 3 and 4. Furthermore, neither 3 nor 4 can be undertaken unless one of the first two opportunities is undertaken. There are no such restrictions on investment opportunities 5 and 6. The objective is to select the combination of capital investments that will maximize the total estimated long-run profit (net present value).

- (a) Formulate a BIP model for this problem.
(b) Use the computer to solve this model.



47. (HL¹, pg. 55) *Union Airways* is adding more flights to and from its hub airport, and so it needs to hire additional customer service agents. However, it is not clear how many more should be hired. Management recognizes the need for cost control while also consistently providing a satisfactory level of service to customers. Therefore, the OR team is studying how to schedule the agents to provide satisfactory service with the smallest personnel cost.

Based on the new schedule of flights an analysis has been made of the minimum number of customer service agents that need to be on duty at different times of the day to provide a satisfactory level of service. The rightmost columns of the following table show the number of agents needed for the time periods given in the first column.

Time Period	Time Periods Covered					Minimum number of Agents Needed
	Shift					
	1	2	3	4	5	
6:00 – 8:00	✓					48
8:00 – 10:00	✓	✓				79
10:00 – 12:00	✓	✓				65
12:00 – 14:00	✓	✓	✓			87
14:00 – 16:00		✓	✓			64
16:00 – 18:00			✓	✓		73
18:00 – 20:00			✓	✓		82
20:00 – 22:00				✓		43
22:00 – 24:00				✓	✓	52
0:00 – 6:00					✓	15
Daily cost per agent	\$170	\$160	\$175	\$180	\$195	

Each agent works an 8-hour shift, five days per week. Checkmarks in the main body of the table show the hours covered by the respective shifts. For each shift the daily compensation (including benefits) for each agent is shown in the bottom row. Determine how many agents should be assigned to the respective shifts each day to minimize the total personnel cost for agents while meeting the service requirements.

48. A company is preparing its investment plan for the next three years. From the current activities predicts the availability of the following funds (in *m.u.*): 50 in year 1; 40 in year 2; 30 in year 3. Beyond this self-financing, the company may seek loans from short-term (one year) in each of the three years. The amount requested in each year shall be paid in the next year along with interest, which is 6% per year. The company can also get in the 1st year a medium term loan to be paid in four years (at the end of the planning period), and each year the interest, which is 7% per year. For financial reasons, the company cannot have for both short-term loans in years 1 and 2. The funds available, including any loans can be applied in the following investment projects:

	cash-flow			TPV* (end of year 3)
	year 1	year 2	year 3	
project 1	-100	20	30	100
project 2	-50	-40	50	90
project 3		-50	50	20

* TPV – Terminal Present Value, that is, the present value at the end of year 3 of the *cash-flows* that will occur in the 4 coming years.

The company intends to maximize the net assets of liabilities at the end of the planning period, that is, intends to maximize the TPV projects less debt at the end of the planning period. Knowing that the projects can only be done in whole, establish a model for this problem.

¹ Hillier and Lieberman, “Introduction to Operations Research”, 9th ed., McGraw-Hill 2010.